$$y = \frac{H_R^2\eta_R}{Y_R}\Delta\eta - \left(\frac{H_R^4\eta_R^2\cos\omega}{2Y_R^3} - \frac{H_R^2}{2Y_R}\right)\Delta\eta^2 + \Bigg[\frac{H_R^6\eta_R^3\cos^2\omega}{2Y_R^5} - \frac{H_R^4\eta_R\cos\omega}{2Y_R^3} + \frac{4}{3}\frac{H_R^5H_R'\eta_R^3\sin\omega}{Y_R^4} - \frac{H_R^6\eta_R^3\sin^2\omega}{2Y_R^5} - \frac{H_R^4(H_R')^2\eta_R^3}{Y_R^3} - \frac{H_R^5H_R''\eta_R^3}{3Y_R^3} + M_R^2\left(\frac{H_R^5H_R''\eta_R^3}{3Y_R^3} + \frac{H_R^4(H_R')^2\eta_R^3}{3Y_R^3} - \frac{2H_R^5H_R'\eta_R^3\sin\omega}{3Y_R^4} + \frac{H_R^6\eta_R^3\sin^2\omega}{3Y_R^5}\right)\Bigg]\Delta\eta^3 \quad (21)$$

$$\frac{M^*}{M_R^*} = 1.0 + \frac{H_R^2\eta_R^2}{Y_R^4}[H_RH_R''Y_R^2 + (H_R')^2Y_R^2 - 2H_RH_R'Y_R\sin\omega + H_R^2\sin^2\omega]\Delta\eta^2 + \Bigg[\frac{H_R^2(H_R')^2\eta_R}{Y_R^2} + \frac{H_R^3H_R''\eta_R}{Y_R^2} + \frac{H_R^4\eta_R\sin^2\omega}{Y_R^4} - \frac{8H_R^4(H_R')^2\eta_R^3\cos\omega}{3Y_R^4} - \frac{4H_R^5H_R''\eta_R^3\cos\omega}{3Y_R^4} - \frac{2H_R^3H_R'\eta_R\sin\omega}{Y_R^3} + \frac{16H_R^5H_R'\eta_R^3\cos\omega\sin\omega}{3Y_R^5} - \frac{8H_R^6\eta_R^3\sin^2\omega\cos\omega}{3Y_R^6}\Bigg]\Delta\eta^3 \quad (22)$$

$$\theta = \frac{H_R^2\eta_R}{Y_R^2}\left(\frac{2H_R'Y_R}{H_R} - \sin\omega\right)\Delta\eta + \Bigg[\frac{H_R^2\eta_R^2\cos\omega}{Y_R^4}\times\left(\frac{3H_R^2\sin\omega}{2} - 2H_RH_R'Y_R\right) + \frac{H_RH_R'}{Y_R} - \frac{H_R^2\sin\omega}{2Y_R^2}\Bigg]\Delta\eta^2 + \Bigg[-\frac{2H_R^3H_R'\eta_R\cos\omega}{Y_R^3} + \frac{3H_R^4\eta_R\sin\omega\cos\omega}{2Y_R^4} - \frac{5H_R^6\eta_R^3\cos^2\omega\sin\omega}{2Y_R^6} + \frac{3H_R^5H_R'\eta_R^3\cos^2\omega}{Y_R^5} - \frac{2H_R^3(H_R')^3\eta_R^3}{3Y_R^3} - \frac{5H_R^4H_R'H_R''\eta_R^3}{3Y_R^3} + \frac{8H_R^4(H_R')^2\eta_R^3\sin\omega}{3Y_R^4} - \frac{10H_R^5H_R'\eta_R^3\sin^2\omega}{3Y_R^5} + \frac{4H_R^5H_R''\eta_R^3\sin\omega}{3Y_R^4} + \frac{4H_R^6\eta_R^3\sin^3\omega}{3Y_R^6} - \frac{H_R^5H_R'''\eta_R^3}{3Y_R^3} + M_R^2\left(\frac{4H_R^3(H_R')^3\eta_R^3}{3Y_R^3} - \frac{13H_R^4(H_R')^2\eta_R^3\sin\omega}{3Y_R^4} + \frac{7H_R^4H_R'H_R''\eta_R^3}{3Y_R^3} + \frac{14H_R^5H_R'\eta_R^3\sin^2\omega}{3Y_R^5} - \frac{5H_R^5H_R''\eta_R^3\sin\omega}{3Y_R^4} - \frac{5H_R^6\eta_R^3\sin^3\omega}{3Y_R^6} + \frac{H_R^5H_R'''\eta_R^3}{3Y_R^3}\right) + M_RM_R'\left(\frac{2H_R^4(H_R')^2\eta_R^3}{3Y_R^3} + \frac{2H_R^5H_R''\eta_R^3}{3Y_R^3} - \frac{4H_R^5H_R'\eta_R^3\sin\omega}{3Y_R^4} + \frac{2H_R^6\eta_R^3\sin^2\omega}{3Y_R^5}\right)\Bigg]\Delta\eta^3 \quad (23)$$

$$A = \pi(Y^2 - Y_R^2)/\cos\omega \quad (28)$$

$$H_R^2 = \{[1 + C_1(1 - e^{[-(\xi-S)^2/2R_SC_1]})]^2\cos\omega + 2[1 + (C_1 - C_1e^{[-(\xi-S)^2/2R_SC_1]})][Y_0 + \xi\sin\omega]\}/\{[1 + C_1(1 - e^{-S^2/2R_SC_1})]^2\cos\omega + 2[1 + C_1(1 - e^{-S^2/2R_SC_1})]Y_0\} \quad (35)$$

The discrepancies in Eqs. (18) and (21) occur in the coefficients of the $\Delta\eta^2$ terms. Equations (20–23) yielded discrepancies in the coefficients of the $\Delta\eta^3$ terms.

Reference

[1] Hopkins, D. F. and Hill, D. E., "Transonic Flow in Unconventional Nozzles," *AIAA Journal*, Vol. 6, No. 5, May 1968, pp. 838–842.

Comments on "Effects of a Dynamic Gas on Breakdown Potential"

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Gardner has presented some experimental results[1] showing that as the flow velocity in a channel is increased, the breakdown potential decreases from its no-flow value and then becomes nearly constant. We shall present an argument that explains, qualitatively, these data.

Since breakdown of a gas is said to occur when the electron density reaches a certain value, the classical calculation of breakdown essentially amounts to an accounting of the electron loss and production mechanisms. Forced convection appears to be a further loss mechanism; consequently the dynamic gas effect is expected to result in an increase rather than a decrease in the breakdown potential. The actual situation is somewhat more complicated for two reasons. First, the electric field in Gardner's experiment seems to be nonuniform, being larger near the electrodes (which constitute two walls of the channel) than near the center of the channel. Second, the distribution of the electrons is different in windoff and windon situations. Recent calculations[2] have shown that, in the presence of forced convection near electrically conducting walls, the electron density profile is fuller; that is, there is a greater electron density. That this is true follows from the fact that forced convection increases electron transport to the wall (as well as heat transfer, etc.). The electron transport is equal to the product of the diffusion coefficient and the normal gradient of number density. Since the diffusion coefficient is constant, the increase in transport is due to the increased gradient, which in turn implies that the profile must be fuller in a convecting fluid.

The production of electrons over a fixed distance is proportional to the product of the number density of electrons with the number of ionizing collisions per electron in this distance. [This term is essentially Townsend's first ionization coefficient (α).] The basic problem is to calculate the electric potential required to increase the electron density to some fixed level. The nonuniform field is such that α is largest near the electrode. Thus, forced convection offers the possibility of reducing breakdown potential by increasing the number of electrons in a high-voltage region as long as the increased losses due to forced convection do not increase as fast as the rate of production of electrons increases. Reference 2 shows that when the electromotive force decreases quadratically with distance away from the wall, the effect of forced convection is to reduce the breakdown voltage around a slot antenna. A detailed calculation for the direct current case depends upon the electric field distribution but also upon

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electrode characteristics and is more complicated than the high-frequency breakdown calculation. Nevertheless the arguments presented here show that Gardner's results, surprising as they initially appear, are quite plausible.

References

[1] Gardner, J. A., "Effects of a Dynamic Gas on Breakdown Potential," *AIAA Journal*, Vol. 6, No. 7, July 1968, pp. 1414–1415.

[2] Covert, E. E., "An Estimate of the Effects of Forced Convection on Antenna Breakdown of Slot Antennas," Rept. 154, August 1968, M.I.T. Aerophysics Laboratory.

Errata and Addenda: "Theoretical Considerations of Panel Flutter at High Supersonic Mach Numbers"

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[AIAA J. 4, 1257–1266 (1966)]

An error in the article has been pointed out by Ellen.[1] Equation (50) should obviously read,

$$f = g_{T2}/g_{T1} = [1 + \psi(g_{S1}/g_A)]/[1 + (g_{S1}/g_A)] \quad (50)$$

In Fig. 16, the ordinates f should be relabelled as $1/f$. It is to be noted that this error would not have any effect on the λ_F for small g_T since the factor $2[f]^{1/2}/(1+f)$ equals $2[1/f]^{1/2}/(1 + 1/f)$, but it may have an effect for large g_T, as can be seen from Eq. (48). Accordingly, the results in Fig. 17 for $g_A = 0.1$ are correct, whereas those for $g_A = 1.0$ may be in error.

In the discussion of the effects of arbitrary structural damping in Sec. 5, the damping in the structure was assumed to come from the general form, $G\partial^{n+1}w/\partial t\partial x^n$ and not from the $(1 + ig)Kw$ form used in conventional V-g modal analyses of panel and aircraft flutter. Thus, the g_i there actually represent critical damping ratios $2\zeta_i$ and *not* the "structural" damping values g_i of the V-g analyses. This has led to some confusion in the article. A brief review of the relationship between the structural damping g_i and the critical damping ratio ζ_i is presented below in hopes of clarifying this Sec. 5.

For the structural damping as used in conventional analyses, one assumes,

$$\rho_M h\partial^2 w/\partial t^2 + D(1 + ig)\partial^4 w/\partial x^4 = 0 \quad (1)$$

for a two-dimensional panel with no air forces acting. Setting $w = q_n(t) \sin n\pi x/a$, one obtains

$$d^2q_n/dt^2 + \omega_n^2(1 + ig)q_n = 0 \quad (2)$$

where the natural frequency ω_n of the nth mode is

$$\omega_n = (n\pi)^2[D/\rho_M ha^4]^{1/2} \quad (3)$$

In complete sinusoidal form, Eq. (2) becomes

$$[-\omega^2 + \omega_n^2(1 + ig)]q_n = 0 \quad (4)$$

One sees from Eq. (4) that the structural damping g_n of each mode ω_n is the same, i.e., $g_n = g$, for the damping approximation assumed in Eq. (1).

For damping type A, one assumes

$$\rho_M h\partial^2 w/\partial t^2 + G_A\partial w/\partial t + D\partial^4 w/\partial x^4 = 0 \quad (5)$$

Setting $w = q_n(t) \sin n\pi x/a$, one obtains

$$d^2q_n/dt^2 + 2\zeta_n\omega_n dq_n/dt + \omega_n^2 q_n = 0 \quad (6)$$

where ω_n is as before and the critical damping ratio ζ_n is given as

$$\zeta_n = G_A/2\rho_M h\omega_n \quad (7)$$

Since G_A is assumed constant in Eq. (5), one finds from Eq. (7) that

$$\zeta_n = \zeta_1\omega_1/\omega_n \quad (8)$$

for the damping type A approximation given in Eq. (5). In sinusoidal form, Eq. (6) becomes

$$[-\omega^2 + \omega_n^2(1 + i2\zeta_n\omega/\omega_n)]q_n = 0 \quad (9)$$

Comparing this with Eq. (4) and using Eq. (8), the equivalent structural damping for this damping type A case would be

$$g_n = 2\zeta_n\omega/\omega_n = 2\zeta_1(\omega_1\omega/\omega_n^2) \quad (10)$$

During flutter, ω = const for all modes; hence $g_2/g_1 = \frac{1}{16}$. Thus a conventional $V - g$ modal analysis with $g_2/g_1 = \frac{1}{16}$ would be equivalent to the damping type A presented here.

For damping type B, one assumes

$$\rho_M h\partial^2 w/\partial t^2 - G_B\partial^3 w/\partial t\partial x^2 + D\partial^4 w/\partial x^4 = 0 \quad (11)$$

Proceeding as in the damping type A case gives

$$\zeta_n = \zeta_1 \quad (12)$$

for the damping type B approximation given in Eq. (11). The equivalent structural damping for this case would be

$$g_n = 2\zeta_n\omega/\omega_n = 2\zeta_1\omega/\omega_n \quad (13)$$

During flutter, ω = const for all modes; hence $g_2/g_1 = \frac{1}{4}$.

For damping type C, one assumes

$$\rho_M h\partial^2 w/\partial t^2 + G_c\partial^5 w/\partial t\partial x^4 + D\partial^4 w/\partial x^4 = 0 \quad (14)$$

Proceeding as in the damping type A case gives

$$\zeta_n = \zeta_1\omega_n/\omega_1 \quad (15)$$

for the damping type C approximation given by Eq. (14). The equivalent structural damping for this case would be

$$g_n = 2\zeta_n\omega/\omega_n = 2\zeta_1\omega/\omega_1 \quad (16)$$

During flutter, ω = const for all modes; hence $g_2/g_1 = 1$.

From this brief review, the following observations can be made:

1) Damping type C is equivalent to structural damping for two-dimensional panels without midplane forces. For this type damping, the ζ_n's of each mode are related by $\zeta_n = \zeta_1\omega_n/\omega_1$. Then, $g = 2\zeta_1\omega_{FLUT}/\omega_1$. For three-dimensional panels, these relations are somewhat altered.

2) In Sec. 5 of the article, the so-called $g_2 = g_1/4$ and $g_2 = g_1$ cases are actually the $\zeta_2 = \zeta_1/4$ and $\zeta_2 = \zeta_1$ cases. These correspond to damping types A and B, respectively. This was pointed out to the author by Jordan.[2] Damping type C would show even greater destabilization in Fig. 17 than damping type B.

3) The footnote on p. 1264 of the article is in error. The $g_2 = g_1$ case of the conventional $V - g$ modal analysis is equivalent to damping type C, and *not* type B. This applies only for two-dimensional panels with no midplane forces.

4) Which of the various types damping to use for such continuous structures depends on experimental measurements of the damping behavior of the various modes ω_n.

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